

15.2 Inferences for the slope of the population regression line:

We will do hypothesis tests to determine if the regression equation ($\hat{y}$) is useful to predict y .

Regression equation from sample data:

$$\hat{y} = b_0 + b_1x \quad (\text{calculator: } \hat{y} = a + bx)$$

$\uparrow$ $\uparrow$
 y -intercept slope

The regression line for population data:

$$y = \beta_0 + \beta_1x \quad \text{If } \beta_1 = 0, \text{ regression is NOT useful to predict } y. \text{ All } x\text{-values would give us the same } y\text{-values.}$$

For hypothesis test:

$H_0: \beta_1 = 0$ (regression line not useful)

$H_a: \beta_1 \neq 0$ (regression line is useful)

LinRegTTest

If P-value $\leq \alpha$ Reject H_0
 (use $\hat{y} = ax + b$ to predict y) Yes, regression line is useful

If P-value $> \alpha$ Do not reject H_0
 (use $\bar{y}$, average y -value, to predict y) No, regression line is not useful.

We cannot make predictions for x -values smaller than our smallest x data value or larger than our largest x data value. This is extrapolation and leads to incorrect numbers.

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EX- Can we use hours studied (x =predictor variable) to predict test score (y =response variable)? Use $\alpha = 0.01$

x	10	15	12	20	8	16	14	22
y	92	81	84	74	86	80	84	80

$H_0: \beta_1 = 0$

$H_a: \beta_1 \neq 0$

Test: LinRegTTest

p-value = 0.024 $> \alpha = 0.01$

Reject? Do not reject H_0

Yes or No? No, the regression line does not appear to be useful.

Predict test score (y) for study time of 15 hours (x). $\rightarrow$ Since regression line not useful, use $\bar{y}$

$\bar{y} = 82.5$

Predict test score for study time of 30 hours.

Can't, extrapolation $\rightarrow$ Data: $8 \leq x \leq 22$... can't predict outside of these values.

Regression equation:
 $y = a + bx \rightarrow \hat{y} = 94.9 - 0.8x$

Get there in calc
 STAT, CALC, 2-varStats

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EX: Can we use house size (100's ft.) to predict house price (\$1000's)? $\alpha = 0.01$

Data in calculator

$$H_0: \beta_1 = 0$$

$$H_a: \beta_1 \neq 0$$

Test: LinRegTTest

$$p\text{-value} = 0.006 \leq \alpha = 0.01$$

Reject? Reject H_0

Yes or No? Yes, the regression equation is useful to predict house prices from house size.

Predict house price for a house size of 3400 ft.² ($x=34$)

Since regression equation useful, use it!

$$\hat{y} = 140.1 + 15.9x$$

round to nearest \$1000

$$\hat{y} = 140.1 + 15.9(34) = 680.7 \times 1000$$

$$= 680,700$$

$$= \$681,000$$

Predict house price if size is 4500 ft.² ($x=45$)

Can't, extrapolation $\rightarrow x\text{-values: } 22 \leq x \leq 40$ [22-40]

x data values; 45 is outside of these values

$$\text{Residual: } y - \hat{y} = e$$

y from data value

predicted y from regression equation

Find residual for $x=34$:

$$\text{Residual: } e = y - \hat{y} = 661 - 681 = -20$$

from data

calculated from $\hat{y}$

$$\hat{y} = 140.1 + 15.9x$$

Find and interpret the y -intercept.

140.1 $\rightarrow$ If house size = 0, price is predicted to be 140.1 $\Rightarrow$ \$140,100

Find and interpret the slope.

slope = 15.9 $\rightarrow$ When house size increases by 1 (100 ft.²) price increases by

15.9 $\Rightarrow$ \$15,900.

Will be asked to find and interpret on Exam!